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AN ANCIENT RULE FOR MAKING PORTABLE ALTITUDE SUNDIALS FROM AN 'UNEDITED' MEDIEVAL TEXT OF THE TENTH CENTURY

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The use of altitude dials that worked on the same principle as the cylinder for travellers may be proved for Christian Europe between the tenth and eleventh centuries by the 'Canterbury Pendant': a little portable sundial made of silver and gold found in 1938, during the work of ground levelling of the cemetery inside the cloister of the cathedral of Canterbury. This sundial shows only the third, the sixth and the ninth hour of the day[1] by a series of points along vertical columns assigned to the months of the year.

The existence of other altitude sundials of the same kind seems to be confirmed by the words used by Byrhtferth of Ramsey (*c*. 970 – *c*. 1020) in a passage of his *Manual*: "the sun ascends point by point on the sundial", and again "Observe, O clerk, how the sun ascends point by point on the sundial".[2] To the same monk was attributed, in 1563, the *Glossae* to the *De temporum ratione* of the Venerable Bede (627–735);[3] in these we read: "The point [one of the fractions of one hour] is so called from the word '*pungendo*'; because of this they make some points on the sundials. The *horologium* is a series of hours shown point after point."[4] Hermann the Lame (Hermannus Contractus, 1013–54) described the instrument for the first time in a short but complete treatise.[5]

Then there is a text, which until now has escaped the attention of scholars, that describes in a clear way the construction of an altitude dial of this kind. It is inserted in a little treatise doubtfully attributed to Bede: the *Libellus de mensura horologii*. This brief composition (for convenience hereafter simply *Libellus*) was published for the first time in 1563 in the first tome of the big work on Bede edited by Iohannes Hervagius (Johann Herwagen) from Basel[6] and reprinted in the second half of the nineteenth century in vol. xc of the well known series on patristic Latin texts edited by J. P. Migne.[7] The *Libellus*, in fact, is a collection of different texts (not all on gnomonics) gathered under seven rubrics in this order:

1. The first does not seem to have a title and it starts with: "*Si quem delectat horologium componere*".[8]
2. The second, *Ad meridiem inveniendum*, describes the method of finding the meridian line with 'correspondent altitudes' of the sun, also known as 'Hindu circles'.
3. The third, *Horologium quod contra unumquemque mensem habet ad umbram humani corporis pede singularum horarum diei*, is the drawing of the 'shadow scheme' falsely attributed to Bede. It is represented as a portable instrument.[9]
4. Then follows *Concordia xii mensium*, where the preceding shadow scheme

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is reported in a textual way.[10]

5. The fifth, *De signis et horis xii mensium*, is a diagram of the length of the days, in equal hours, for every month and the corresponding zodiacal sign.[11]

6. Then, *De tribus diebus periculosis* lists the three dangerous days of the year, when one risks death.

7. The last one, *De Aegyptiacis diebus*, is a known text about the so called 'Egyptian days', which lists all the 22 days of the year that are disadvantageous for human activities.[12]

The drawing of item no. 3 and the next text (no. 4) are found often together in a single folio,[13] while the text of no. 2 is added to many medieval gnomonical works independently from the kind of sundial described.

The contiguity of items 2, 3 and 4 has always drawn, automatically, the attention of researchers on 'shadow schemes', which were so popular in medieval times.[14] For this reason the first text of the *Libellus* has not been studied carefully and sometimes the entire compilation has even been mistreated as a disappointing gnomonic work.

We do not know the source consulted by Hervagius for his work, but whatever it was it seems now irreparably lost. For all the sections relating to the *computus*, Charles Jones suggested that Hervagius used a thirteenth-century manuscript, or manuscripts, from southern Germany, but we do not know anything about the manuscript that Hervagius used for his publication of the *Libellus*.[15] I do not know of any author who has been able to find another manuscript with the same compilation as the *Libellus*, nor have I been able to find one.[16] Jones admits that he never met a manuscript with the same title, and because of that he is not able to mention any other manuscript with text no.1; after this remark he suddenly goes to the texts and drawings in 3, 4, 5, 6 and 7, where he is able to cite many manuscripts.[17]

There is the Migne text, and until now no one found a second text. However, I can add two other texts, although I think there might be more.[18] So at this point we know just three sources with text no.1:

A. The first text of the *Libellus*, published first by Hervagius, and then by Migne. It has almost certainly the title that Hervagius gave to the full collection: *Libellus de mensura horologii*. It seems that the manuscript is definitely lost and we do not know its provenance (thirteenth century?).

B. A fragment of only six lines added in a narrow space at the bottom of folio 7v of Ms. Vaticano Pal. 3101 (eleventh century), entitled *Inventio pendentis orologii*.[19]

C. A manuscript written in the eleventh or twelfth century and now kept in the Badische Landesbibliothek of Karlsruhe (Ms. 504 Karlsruhe, fol. 58v). The text is preceded by the title *Ratio ad componendum horologium.*[20]

The Text

I will use A as the basic text for this edition, because the evidence shows that this is surely the most complete and the nearest to the original layout; so I will leave the references to the columns as they are in the text of Migne (which is also easier to find than the Hervagius edition) but I will entitle it as in B, because I think that this is more significant than the too generic titles in A and in C. Moreover I will correct the text following the readings of B and C.

Beda Venerabilis (627–735): *Opera dubia*	Pseudo-Bede
Libellus de mensura horologii	**Manual for drawing a sundial**
Inventio pendentis orologii [21]	Making the pendant sundial
I (C: row 28) (A: col. 951) *Si quem delectat horologium componere, sive metallinum, sive ligneum, quod ad instar*[22] *perpendicularis videtur formari, praeter quod illud rotundum est, istud vero per sena latera, in quibus horarum ordo* (A: col. 952) *totius anni binis mensibus insignitur.* [23]	If anyone wishes to make a sundial, of metal or wood, similar to a perpendicular (sundial), not cylindrical but with six sides on which the hours are marked for the entire year in pairs of months.
II *Longitudo namque quinque digitorum creditur sufficiens esse, plus minusve. Grossitudo quoque, si unumquodque latus sufficit ad gnomonem et inscriptionem Kaleda-* (A: col. 953) *-rum, satis videtur.*	The length (of the sundial) needs to be about five fingers; it needs to be thick enough so that each side can accommodate the gnomon and the names of the pairs of months.
III (B: row 40) *Prima linea quae ascribitur xii Kal*[dis] *Iuni et iulii,*[24] *secundum placitum habeat initium: ipsum tamen spatium sex aequis spatiis dividatur, vel partibus, a primo puncto quod in una regione habetur, usque ad umbilicum. Secunda vero, quae xii Kalend. Augusti et Maii assignatur, a medio sextae horae spatio primi ordinis sortiatur terminum. Tertia, quae Aprilis et Septembris iisdem (ut supra titulatur) Kalendis, relicto dodrante penultimae secundi ordinis horae, quadrantem sumit exordium. Quarta penultimi spatii tertiae seriei, vindicat trientem. Quinta penultimi spatii quarti ordinis, quintam arripit partem. Sextae lineae terminus, quartae horae*[25] *confinio quinti ordinis coaequatur.*	The first line, assigned to the twelfth (day before the) Calends of June [May 21] and July [June 20], has its length beginning where you like: its length is divided into six equal parts, starting from the first point in the section up to the gnomon.[26] The second line, assigned to the twelfth (day before the) Calends of August [July 21] and May [April 20], begins at the point in the middle of the sixth hour space of the first section.[27] The third (line), assigned to the same calends as previously noted of April [March 21] and September [August 21] begins three-quarters of the way through the penultimate hour of the second section. The fourth (line) begins one third of the way through the penultimate hour of the third section. The fifth (line) begins one-fifth of the way through the penultimate hour of the fourth section. The end of the sixth line shares the boundary of the fourth hour of the fifth section.

The text in B is missing the first parts (I and II), which, on the contrary, are present in A and in C; its *incipit* is thus: *Prima linea quae ascribitur xii kalendis Iunii et iulii*. Otherwise, all the texts, A, B and C, are almost identical, with only certain differences that will be discussed in the references.

The Instrument

The first paragraph (I) describes the shape and the kind of the sundial, in our case a pendant altitude dial, apparently with a prismatic shape with hexagonal section (*per sena latera*) working on the same principle as the cylinder dial (Figure 1).[28]

The second paragraph (II) describes the instrument's dimensions. The suggested length for this object is around 5 fingers (about 9 cm) and its thickness (width or diameter) is enough so that every side can hold the gnomon and the writings related to the calendar.

The last part of the text (III) teaches the construction of the month lines and gives the parameters for the relative lengths of the meridian shadows.

The Calendar

Every day line, or monthly column, has assigned two solar months with reciprocal correspondence to the same solar declinations. This match was very common in Roman portable sundials, but in this special case the date of the sun's passage between

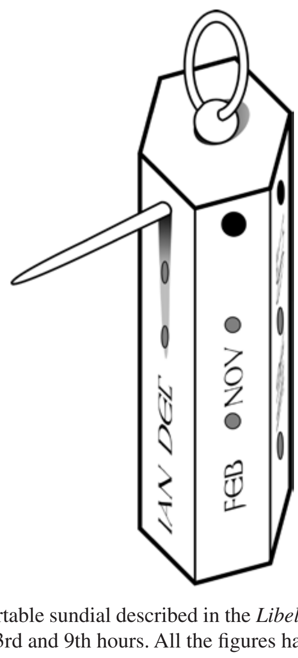

FIG. 1. Reconstruction of the portable sundial described in the *Libellus de mensura horologii.* The points here show only the 6th, 3rd and 9th hours. All the figures have been drawn by the author.

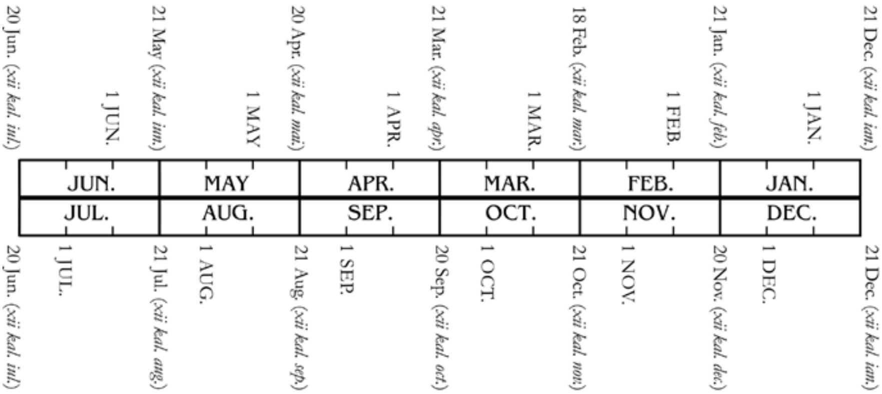

FIG. 2. The calendar suggested by the *Libellus*.

zodiacal signs (12 days before Calends) has a late Roman origin.[29] In a calendar such as the one suggested by the *Libellus*, and reconstructed in Figure 2, the data of the meridian shadow length in every column cannot be taken at the beginning of the solar month, that is, the first day of the sun's passage to the next zodiacal sign (about the 21st of every month); but one must consider it in the middle point between two zodiacal signs passages: in other words, with the solar declination near the fifth day of every month of the year. The data taken in these days will be used by me further when analysing the horary scheme.

Construction of the Hour Points

Every monthly line, once it is drawn with the right length, is subdivided into 6 equal parts that represent both the first six hours of the day (from sunrise to midday) and also the last six hours (from midday until sunset).[30] This division in equal parts appears obviously ingenuous but it is not alone in medieval literature: we can find it in the 21st chapter titled *De inveniendis in dorso Aastrolabii horis* of the first book of the *De utilitatibus astrolabii*, where this division is given as good for making *horologia* (sundials).[31]

The making of the sundial described in the *Libellus* starts with the summer solstitial line, which in our case is assigned to two periods that go from 21 May to 20 June and from there back until 21 July (*xii Kalendis Iuni et iulii*). This line may be as long as one wants (obviously as long as possible). The extreme point of the second line, which corresponds to the period from 21 July up to 21 August and from 20 April until 21 May (*xii Kalend. Augusti et Maii*), matches with the middle point of the sixth space (that is, the space corresponding to the 6th and 7th hours) in the preceding months. The third line, dedicated to the 12th day before calends of April and September (21 March and 21 August), starts from 3/4 of the penultimate space (between the 5th and 8th hours) of the preceding month's line. The fourth line starts

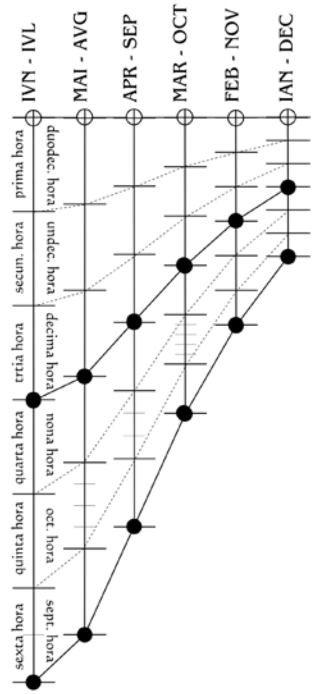

FIG. 3. Scale of the lines drawn according to the manuscripts of the *Libellus de mensura horologii*. In the present sketch the lines of the sixth and the third/ninth hours are shown with dots.

from 1/3 of the penultimate space of the preceding month line. The fifth one, at 1/5 of the penultimate space of the preceding one, and the sixth one ends at the limit of the fourth space (end of the 4th and beginning of the 9th hours) of the preceding column. The result is shown in Figure 3.

The layout found from the data furnished in the *Libellus* unequivocally reminds us of the hour lines of a cylindrical portable sundial that was made in the Middle Ages. It was also known by the name of *horologium viatorum*, and today is more commonly known as 'the shepherd's dial'.

The Cylinder Sundial and Its Origin

It is believed that the oldest Latin document describing the construction of this kind of sundial was compiled in the eleventh century by Hermann the Lame, who was a Benedictine monk from the Abbey of Reichenau, Germany, and author also of an essay on the astrolabe (*De mensura astrolabii liber*) that is believed to have been translated from an Arab treatise because of its frequent use of Arabic technical terms in its contents.[32] The text related to the construction of a cylindrical altitude dial is contained in the second book of the *De utilitatibus astrolabii*.[33] The text on the cylinder written by Hermann looks independent from the text of the *Libellus*. Hermann explains how to construct the instrument from a table of solar altitude data found with an astrolabe. He finds the hour shadow lengths by an easy but correct graphical procedure.[34] The Hermann text on the cylinder dial was very widely diffused, and many other new Latin and European treatises drew on it.

The Arabs also knew a sundial similar to the cylinder dial that was widespread in medieval and Renaissance times in Europe. The principle was the same as for the so-called "locust's leg", a dial made with a flat rectangular board divided into columns for every sun declination or zodiacal sign. In every column or vertical day-line are marked the hours of the day. The gnomon can be either fixed, or movable by being inserted in a series of holes on the top of every column.

This dial could be mounted on a body with a circular section but usually was intended more as a cone dial than a cylinder. The name of both kinds was *mukḥula*. We know that the oldest Arabic text about an altitude dial like this is probably from ninth-century Baghdad.[35] Al-Bīrūnī mentions the *mukḥula* but he does not explain any detail of its construction. Al-Khwārizmī, in the late tenth century, names the *mukḥula* as an horary instrument. We know also that in the thirteenth century an author named Ibn Yayā al-Siqillī wrote a treatise on the *mukḥula*.[36] In addition, al-Marrākushī wrote about it, and then Najm al-Dīn al-Mīsrī in the fourteenth century.[37]

For a time it was believed that Hermann was the inventor of the portable cylinder dial, since his writing on the subject does not use any Arabic terms (as does his essay on the astrolabe). Today we know that neither Hermann the Lame nor the Arabs (as was later supposed for a time) were the inventors of the dial, because a small portable cylinder dial made of bone (height 62 mm, diameter 25 mm) was found at Este in the province of Padua, Italy, in the grave of a Roman doctor that has been dated back at least to the first century. This is the proof that that model was already known in Roman times (Figure 4). [38]

The principle of the 'locust's leg' was also known to the Romans, as we see from the well-known 'Ham of Portici' found at Ercolano, near Naples.[39]

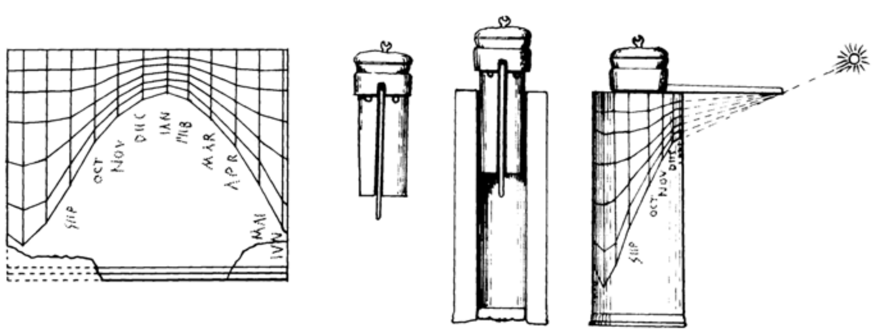

FIG. 4. A cylindrical sundial for travellers found at Este (Padua).

It is clear, therefore, that Hermann drew his information from some earlier scientific tradition but I find it difficult to believe that Hermann looked at an Arabic manuscript, because in his text there is not one single Arabic word, and we do not really know that an Arabic manuscript on the *mukḥula* or on the 'locust's leg', older or even contemporary with him, circulated in Europe in the tenth or eleventh century. I believe that Hermann knew some old Latin-tradition text about it and, on the other hand, that he had some Arabic knowledge or instrument that permitted him to write a new independent and corrected text.

So now, in order to find the tradition followed by the *Libellus*, the questions that we pose are:

> Has the instrument described in the *Libellus* some relationship to the Roman cylinder found at Este?
>
> Does the layout produced by the data furnished by the *Libellus* correspond to a correct course of the hours or is it simply an ingenuous approximate attempt?
>
> If its layout is correct, for which latitude was it calculated or it was considered universal?

To answer these questions we must compare the drawing produced from the *Libellus* with the two most important testimonies of the past: the Roman dial from Este and the Anglo-Saxon dial from Canterbury.

The Cylinder of Este and the Libellus's Horologium: Analysis and Comparisons of the Layouts

Nothing is written in the *Libellus* text about the length of the gnomon, so we need to hypothesize some length to start our investigation. Of course every length used for testing is arbitrary, nevertheless we must start with some reasonable choice.

The dimensions of the dial given in the *Libellus* are very similar to the proportions of the Este cylinder dial, so I have tried a length of a gnomon of about 20 mm, comparable to the length of the summer gnomon of Este,[40] and calculated an hourly layout for different meaningful latitudes: 45°N,[41] 48°N,[42] 51°N[43] and 53°N. None of these four latitudes succeeds in having a correct overlap with the hourly course furnished by the *Libellus*. In all these four cases the errors are worse in the two solstitial periods, mostly in JAN–DEC for latitudes higher than 45° and in JUN–JUL for latitudes lower than 51°. See Figure 5.

The shadow of the gnomon fits well only at the summer solstice for latitude 51°N and at the winter solstice for latitude 45°N or a little lower. Changing the length of the gnomon does not solve the dilemma. It is evident that the problem must be tackled from another point of view.

Has a purely intuitive method perhaps been used, not mathematical and even not empirical, to build the resultant meridian curve from the rules of the *Libellus*?

If there is a relationship between the first text of the *Libellus* and the hourly curves of the cylinder of Este, this is not to be sought in the relationships among the two

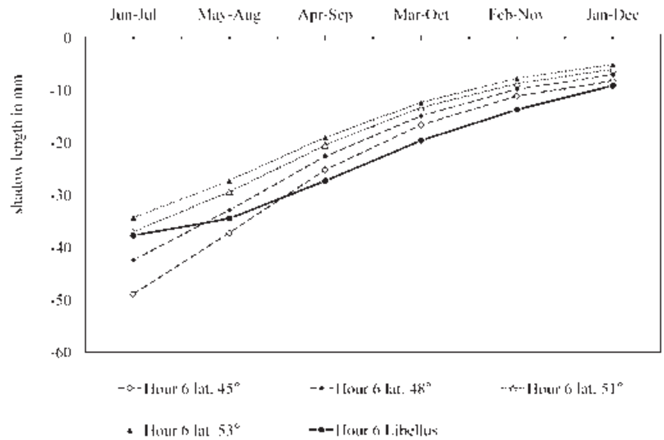

FIG. 5. Comparison of the noon line (sixth hour) data of the *Libellus* dial with the same hour line correctly calculated for latitudes 45°N, 48°N, 51°N and 53°N with gnomon length of 20 mm.

meridian shadow lengths, because the curve of the sixth hour on the cylinder extends in height much more than the one described in the *Libellus* (Figure 6).

If there is a connection (and I believe that there is), this is to be sought partly in the same type of 'levelling' of the meridian hourly curve. An hourly curve so flattened, in effect, can be produced, as far as I know, only in three ways: (1) with a fixed gnomon (e.g., the 'Ham of Ercolano', better known as the 'Ham of Portici'), (2) with a conical body, or (3) with the seasonal use of two, three or more gnomons of different lengths.

The first case is to be discarded because in the text of the *Libellus* it is clear that it

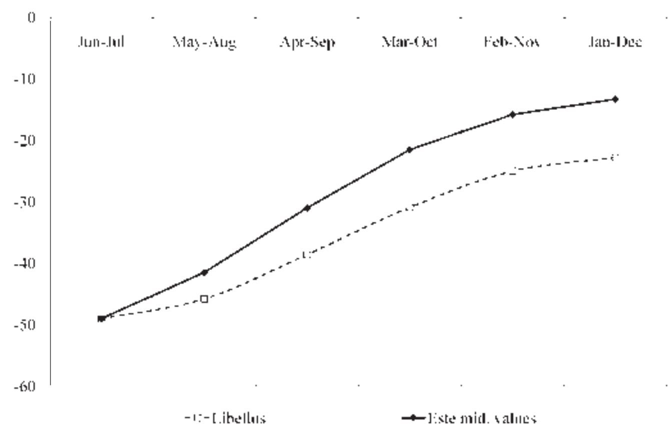

FIG. 6. Comparison of the noon curve on the cylinder of Este and the midday curve described in the *Libellus de mensura horologii*. I put the starting point at the shadow length of the summer solstice.

intends that the gnomon is to be put from time to time on the correct monthly column ("*Grossitudo quoque, him unumquodque latus sufficit ad gnomonem et inscriptionem Kaledarum, satis videtur*"). The second one is doubtful, because in the *Libellus* this idea is not mentioned: we only read that it is similar to the cylindrical ones. Nevertheless I have tried to test this possibility without appreciable success. Therefore only the third suggestion — the use of two or more different gnomons — is feasible. The 'cylinder of Este' actually has two gnomons: one for the summer period (the shorter one, 21 mm) and one for the winter period (27 mm).[44] Calculation has allowed us to confirm the 'acceptable approximation' (we should not forget that all the portable sundials of the Antiquity were of very small dimensions) of these hourly curves for the latitude of Este (Figure 7).[45] However, in the graph produced by calculation (white squares) a significant jump occurs at the equinoctial period; this sudden leap of the curve is due to the change of gnomon length.

With the same procedure we discover that the curve for the sixth hour (noon) calculated on the data furnished by the manuscript of the *Libellus*, besides showing an evident relationship with the same curve of the cylinder of Este, belongs also to a layout produced by two gnomons of around 20 and 25 mm, for a latitude near to 50°N (Figure 8).

The Canterbury Pendant

Besides the portable cylinder of Este, the second most famous testimony of the presence of portable altitude sundials is the object known today as the 'Canterbury pendant'. The pendant has been dated by scholars to the tenth century (before the Norman Conquest of England in 1066), therefore still in the Anglo-Saxon epoch.[46]

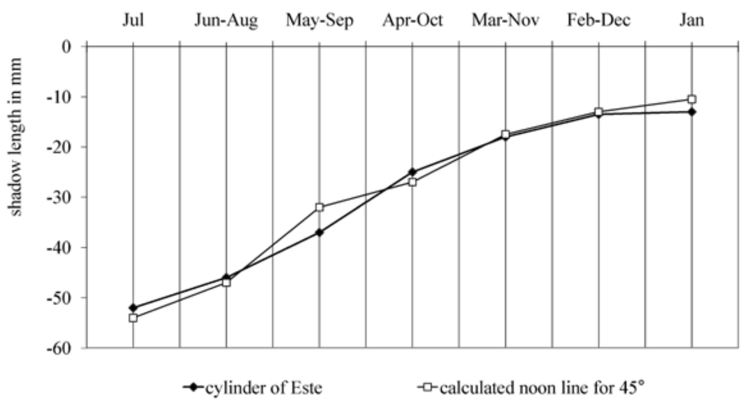

FIG. 7. Comparison of the noon curve of the Este cylinder and the one calculated for latitude 45°N with two gnomons, the shorter one (21 mm) for the summer months (July, June-August, May-September and April) and the longer one (27 mm) for the winter months (January, February-December, March-November and October).

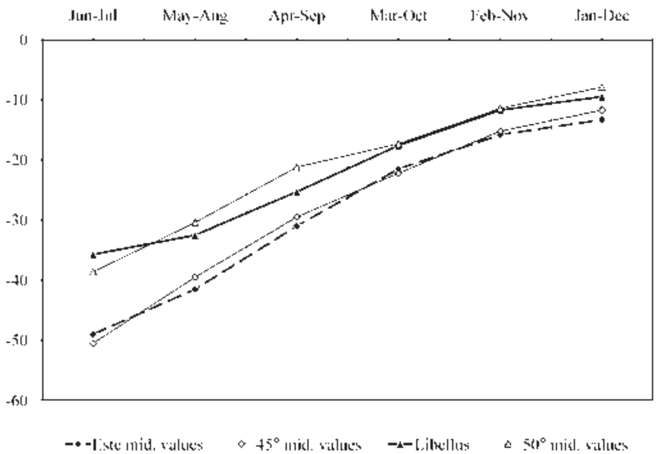

FIG. 8. Comparison of the noon lines of the cylinder of Este and the *Libellus*. In both cases the curves show a compromise aesthetically smoother than the correctly calculated hour curves.

The instrument is mainly a rectangular plate with plain surface, 65 mm tall, 19 mm wide and 5 mm thick; we can think of it as a flat cylinder. The gnomon, which is separate, is decorated at one of its ends with a small dragon head or snake, with a sphere (perhaps the sun) in its jaws, and it measures 30 mm in length.[47] Each of the two faces of the dial is divided into three rectangular columns, each column related to two of the months as described in the text of the *Libellus*. At the top of every column a hole is provided for the insertion of the gnomon, whose shadow vertically extends down to show the hours, reaching some marks on its surface (Figure 9).[48]

We have just seen that the curve of the *Libellus* corresponds to the layout calculated for the latitude of 50°N, a latitude very near that of Canterbury. Is it possible that there is a connection with the pendant of Canterbury and the text of the *Libellus*?

I have compared the graph of the hours drawn by the *Libellus* with the course of the hourly points on the Canterbury pendant. Following the method described in the manuscripts that we are studying, I have started from the real length of the line of the summer solstice in the Canterbury pendant (37.7 mm) and, as can be seen in Figure 10, despite the zigzag course of the hourly points on the pendant, the two layouts remain roughly within common limits.

A close relationship to the scheme furnished by the *Libellus* and the hourly points engraved on the Canterbury pendant is therefore more than evident but still not fully satisfactory. Why in the Canterbury dial do the spots run so zigzagged, and not smoothly as we can see in the *Libellus* or in the cylinder of Este?

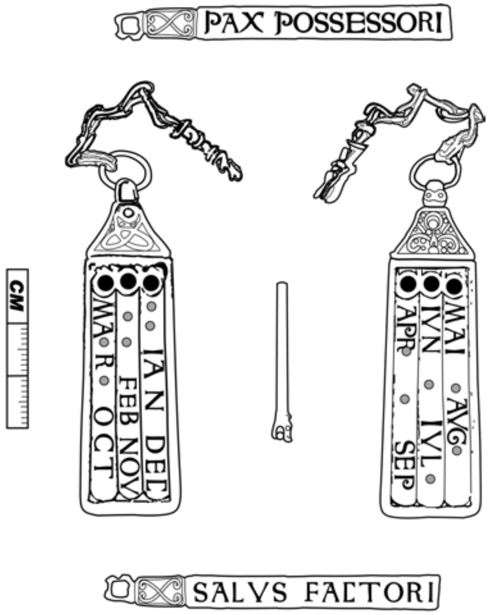

FIG. 9. Drawing of the portable sundial found at Canterbury in 1938.

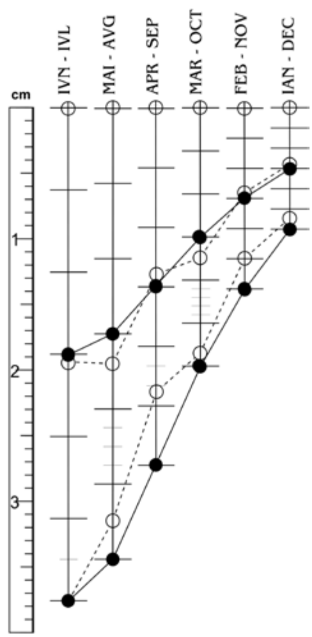

FIG. 10. Comparison of the shadow scale in the *Libellus* (black spots) with the shadow scale on the Canterbury dial (white spots).

Two Gnomons

A study of the Canterbury pendant has been made more than once by different researchers[49] and all have faced the problem of the scheme of the hours when analysing the instrument; each has proposed his own solution. The conclusion that all have reached, good or bad, is that from such a small object and for the epoch in which was built, one cannot achieve greater precision. The strange zigzag course of the hourly points of the pendant has been considered with the indulgence that is granted to a baby, since the Canterbury sundial must have kept only a rough approximation to the time.

To this point it is really that zigzag course, seen already in the layout produced with the calculation on the cylinder of Este (Figure 7), that forces us to reconsider the hourly layout of the Canterbury pendant under a new light: the presence of two gnomons.

It can be seen that the Canterbury pendant possesses only one gnomon, for there is only a single storage hole on the bottom of the plate;[50] but the photographs produced by Binns[51] and by Mills[52] show evidence that the same gnomon could be used in two different ways. Binns uses the gnomon taking the head of the dragon as his vertex, therefore leaving 25 mm of it out of the hole on the dial plate. Mills realizes that the stylus can easily pass through the holes at the top of every monthly column, and by pushing it through the plate until the dragon head stops it, the gnomon projects 20 mm on the opposite face of the dial (see Figure 11).

Using this double position we will get for the winter months a gnomon of 25 mm[53] and for the summer months a gnomon of 20 mm.[54] The calculation shows that, contrary to what has been believed until now, the Canterbury pendant was built correctly for the latitude (or, better, for the 'climate') of that city, and that it actually worked

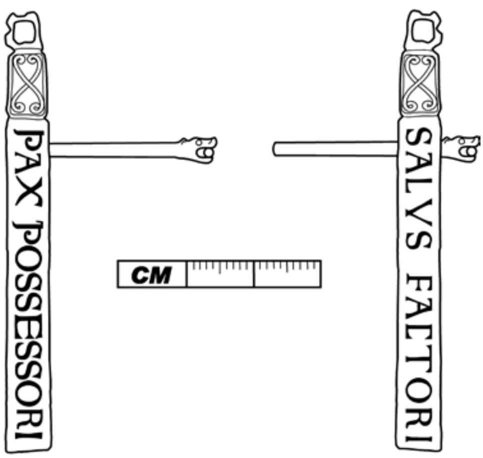

FIG. 11. The double position of the gnomon in the Canterbury pendant.

with two gnomons. The result can be seen in Table 1 and in the graph in Figure 12.

Still, in Figure 12, the only line that lies correctly is that for the sixth hour, while the lines for the third and ninth hours do not correspond very well with the results of accurate calculation. Comparing Figures 3, 10 and 12, it appears evident that the method of hourly subdivision over the sixth hour is the same as proposed by the *Libellus*.

ArabicTreatises

As far as I know, in all *mukḥula* descriptions the gnomon is movable and the body section is rounded, but in the fourteenth-century manuscript of Najm al-Dīn we can find some correspondence to the *Libellus* text. Najm al-Dīn uses a curious expression to define the dial in his manuscript. He writes that it was "a flat conical sundial".

TABLE 1. Comparison of the hour lines 3/9 and 6 on the Canterbury pendant with the same hours calculated for latitude 50°N with two gnomons — the shorter one (20 mm) used for the summer months (April-September, May-August, June-July) and the longer one (25 mm) for the winter months (January-December, February-November, March-October).

Months	Latitude 50° Gnom. Length 20–25mm		Canterbury Pendant Data	
	3 & 9	6	3 & 9	6
Jun–Jul	–14.7	–38.6	–19.5	–37.7
May–Aug	–13.8	–30.4	–19.5	–31.5
Apr–Sep	–11.7	–21.2	–12.7	–21.7
Mar–Oct	–11.1	–17.3	–11.5	–18.7
Feb–Nov	–7.9	–11.4	–6.6	–11.5
Jan–Dec	–5.7	–7.9	–4.2	–8.5

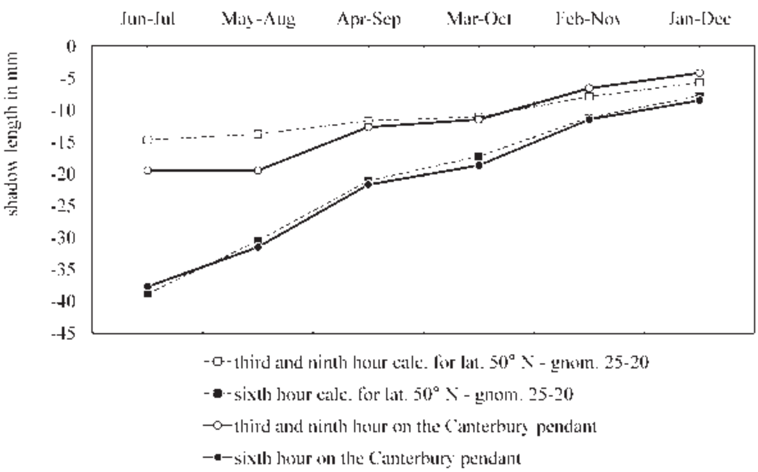

FIG. 12. Graphical display of the data in Table 1. Comparison of the hour lines 3/9 and 6 on the Canterbury pendant with the same hours calculated for latitude 50°N with two gnomons.

What the author meant by that expression can be now understood by comparing this with the *Libellus* text, which describes the instrument as "similar to the one that is perpendicular, bearing in mind that that one is rounded and this one has six sides". So we can understand that the *mukḥula* described by Najm al-Dīn does not have a rounded section but is more similar to a pyramid with six or twelve sides. In the same chapter we can find also the same division of the summer solstice column in six parts.[55]

The *Libellus* text seems to be lacking the part concerned with the gnomon's (or gnomons') length. Notwithstanding, it is demonstrated that the midday curve has been calculated for the use of two gnomons.

In the thirteenth-century treatise written by ibn Yahyā al-Siqillī[56] and edited by Wiedemann and Würschmidt we read:

> Know that in the head of the dial are three sticks.[57] If the shortest of them in the region where you are is too short take the longer one, that is, if the shadow of the shortest stick does not reach the line for the midday.… Operate with the stick of which the shadow of this day falls on the midday line as long as you are in that region. If none of the three sticks fits this line and all of them are too long take the shortest and shorten it until the shadow is the right length. If all of them are too short take the longest of them and stretch it a little bit until it is the right length. Do similarly in every region where you stop and use the method for setting the midday time in the whole region where you travel through.[58]

This passage reveals the possibility of using two or more gnomons to fit the shadow cast on the midday point. This method is incompetently used by ibn Yayā al-Siqillī to make the instrument adaptable for different latitudes. The problem is that a change of latitude changes also the graph of the hour curves, so the only possibility to work in an acceptable way is to draw an approximate curve; but nevertheless this is suitable enough for a small range of latitudes.[59]

The Libellus, Este and Canterbury: Conclusive Relationships

Everything discussed so far shows that the relationships between the *Libellus de mensura horologii*, the cylinder of Este, and the Canterbury pendant are these:

> 1. The data related to the sixth hour (midday) furnished by the *Libellus* produce an approximate curve with the same characteristics as the curve traced on the portable cylinder of Este but corresponding to a northern latitude of around 50°N. The curve produced envisages the use of two gnomons of different lengths for two seasonal times, as with the cylinder of Este. To the detriment of precision (irrelevant for small dimensions), there is a preference for the elimination of the equinoctial jump for an approximate but more harmonious curve.
>
> 2. The course of the hourly points of the sixth hour on the portable sundial of Canterbury answers sufficiently well to the values furnished by the *Libellus*, but it reveals a preference for the correct zigzag layout, against the fluid elegance furnished by the curve of the *Libellus*.

3. Both in the cylinder of Este and in the Canterbury pendant, as well as in the data furnished by the *Libellus,* the relationship between the two gnomons is around 4 to 5.

4. The course of the hourly points of the third and ninth hours in the Canterbury pendant seems to follow the same subdivision in six equal parts described in the *Libellus*, while the cylinder of Este follows a more correct method.

Possible Origin of the Libellus

As we have so few sources, it is not easy to date the text of the *Libellus*; the most ancient manuscript among those listed at the beginning of this work is of the eleventh century. Nevertheless, from the evidence found during the study of the text, we can reasonably suppose that we are dealing with a more ancient tradition. If we consider that the compilation that has reached us by Hervagius is composed of elements, some known since Antiquity (texts 2, 6 and 7), some dated back to the eighth century (3 and 4),[60] and others at least to the eighth century (5), we could date back the first text of the *Libellus* at least to the tenth century, if not earlier to Carolingian times.

There are, actually, two rather clear references to traditions more ancient than the tenth century: the first one can be recognized in the use of the word *umbilicus* for identifying the gnomon,[61] and the second one can be seen in the calendar limits (*xii kalendae*) that became conventional after the Council of Nicea (A.D. 325).

Apart from in Pliny the Elder, the definition of the gnomon with the name *umbilicus* is also found in Bede's *De natura rerum*, which admittedly gets much from Pliny, while the same position of the dates in reference to the months is found in the description of a *horologium* built according to special 'shadow schemes' in a manuscript of the year 850 (Ms Cotton Tiberius Á., III, fols 178–9).[62]

We can find also that Hermann the Lame composed his *Demonstratio componendi cum convertibili sciothero horologeci viatorum instrumenti* looking at a text from the same tradition (perhaps lost today) because he uses the Greek-origin word *sciotherum* as 'gnomon'. Before Hermann we find that word only in the ancient texts that came from Roman times up to Carolingian times.[63]

In conclusion, it seems that the text of the *Libellus*, as we know it today, was probably composed during the eighth to tenth centuries, but that it has a great affinity with the 'cylinder of Este' layout. It is in reality the last witness of the Roman handbooks devoted to the construction of the portable sundials of which Vitruvius also speaks and that it was still probably available up to the beginning of the Middle Ages.[64]

With this conclusion, we can imagine that Roman popular manuals on construction of portable sundials were probably composed by use of tables of midday shadow length calculated for different 'climates'.[65] The *Libellus* seems to be a surviving fragment related to England's 'climate', transcribed in Carolingian times, when it was perhaps taken as a 'universal' scheme for portable sundials of that kind.

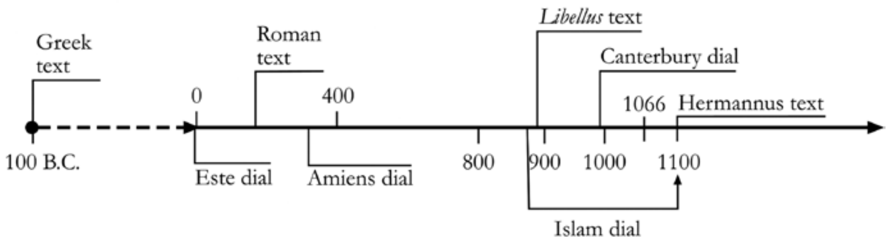

FIG. 13. Time line.

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REFERENCES

1. These were temporary or seasonal hours that were used in the Greco-Roman epoch and into the Middle Ages. The day was divided into twelve parts from sunrise to sunset, and likewise the night from sunset to sunrise. The sixth hour of the day was noon, and the sixth hour of the night was midnight. The length of the day hour therefore differed from day to day, being shorter in summer and longer in winter.
2. "seo sunne astihð pricmælum on þam dagmæle", "Hawa, la cleric, hu seo sunne pricmælum stihð on þam dagmæle", S. J. Crawford, *Byrthfert's manual (A.D. 1011)* (London, 1929), 114–15. See also Peter S. Baker and Michael Lapidge, *Byrhtferth's Enchiridion* (London, 1995).
3. On the contradictory work of Herwagen (Hervagius) see Charles W. Jones, *Bedae pseudoepigrapha: Scientific writings falsely attributed to Bede* (New York, 1939), 14–18; reprinted in Charles W. Jones, *Bedae, the schools and the computus* (Aldershot, 1994). On the false attribution of the *Glossae* to Byrhtferth, see Charles W. Jones, "The Byrhtferth Glosses", *Medium aevum*, vii (1938), 81–97, and Jones, *Bedae pseudoepigrapha*, 21 et seq.
4. "*Puntus a pungendo est dictus, eo quod quibusdam punctionibus certae designationis in horologii designetur. Horologium, id est, series horarum de puncto scilicet in punctum*", Byrhtferth, *Glossae et scholia*, in Beda Venerabilis, *De temporum ratione*, in Jacques Paul Migne (ed.), *Patrologia Latina* (217 vols, Paris, 1844–55; hereafter: *PL*), xc, col. 304.
5. Hermannus Contractus, *De utilitatibus astrolabii libri duo*, *Liber secundus, caput primum, Demonstratio componendi cum convertibili sciothero horologeci viatorum instrumenti*, *PL*, cxliii, cols 405–8B.
6. Johann Herwagen (Beda Venerabilis), *Opera Bedae Venerabilis presbitery, Anglosaxonis: Viri in divinis atque humanis literis exercitatissimi: Omnia in octo tomos distincta…*, i (Basel, 1563), cols 464–8.
7. Beda Venerabilis, *Libellus de mensura horologii*, *PL*, xc, cols 951–6.
8. It is possible that Hervagius has misinterpreted the text of the manuscript in his possession and that *Libellus de mensura horologii* is really the title of the first text.
9. The earliest sketches of this type of *horologium* are from the eighth and ninth centuries; see Karlheinz Schaldach, "Gli 'schemi delle ombre' nel Medio Evo latino", *Gnomonica Italiana*, xvi (2008), 9–16.
10. In this case too the earliest text of the *Concordia duodecim mensium* dates back to the eighth century (see Carla Morini, "Horologium e Daegmael nei manoscritti Anglosassoni di computo", *Aevum*, lxxiii (1999), 273–93, p. 286) and even before (Isidorus of Seville, *De natura rerum*, chap. 5,

"De concordia mensium"), but we can recognize the roots of it in more ancient times.

11. This diagram appears together usually with other *rotae* in the manuscripts coming from Fleury. According to Jones, *op. cit.* (ref. 3), the earliest scheme of this type is found in the Ms. Paris BN, Lat. 5543, fols 134v–141r.
12. This text, together with (6), can be read, with slight differences, in many manuscripts and works; see Lynn Thorndike, *History of magic and experimental science*, i (New York, 1923), chap. 29 and appendix 2. The earliest manuscript in which this text can be read, it seems, is Ms. Berlin Staadtbibliotek *Phill*. 1869, fol. 12r (ninth cent.). See Jones, *op. cit.* (ref. 3), 88. Thorndike, *History of magic*, says that the earliest is the ninth-century century ms. Paris, B.N. nouv. acq. 1616, fol. 12r. It is, however, also a well known theme in Antiquity and therefore not purely medieval.
13. See Schaldach, *op. cit.* (ref. 9).
14. On 'shadow schemes', see Schaldach, *op. cit.* (ref. 9).
15. Jones, *op. cit.* (ref. 3), 15.
16. Ms. Berlin Staadtbibliotek *Phill*. 1833, recently consulted by me, quoted by Jones, *op. cit.* (ref. 3), by Arno Borst, *The ordering of time: From the ancient computus to the modern computer* (Cambridge, 1993), and also by Morini, "Horologium" (ref. 10) as the probable source of the sketch of the 'shadow scheme' (3) and of the *Concordia xii mensium* (4), does not contain texts 1, 6 and 7 of the *Libellus*. Although both Jones and Morini write that the manuscript practically presents text 4 with the same title as in the *Libellus*, in reality it is not this way: in the manuscript that table is divided in two parts, each entitled *Concordia sex mensium*.
17. Jones, *op. cit.* (ref. 3), 87.
18. Although I have found no others, I did work hard to find them. In their collection of incipits, Thorndike and Kibre list only A and B, but do not recognize B as a fragment of A (Lynn Thorndike and Pearl Kibre, *A catalogue of incipits of mediaeval scientific writings in Latin* (London and Cambridge, MA, 1963)). I call attention here to A, B and C for the first time.
19. Construction of the suspended sundial.
20. Rule to make a sundial.
21. B: "*Inventio pendentis orologi*" ("the construction of the suspended sundial"). The title of B is, as far as we know, the only one that reveals the exact nature of the sundial described in the first text of the *Libellus*.
22. C: "*quod instar*".
23. C lacks the reference to the cylindrical sundial and we read only: "... *videtur formari, per sena latera, in quibus ordo horarum totius anni*...."
24. I accept here the reading of B ("*xii Kal^{dis} Iuni et iulii*") against that which is certainly wrong of A ("*xii Kal. Januarii atque Julii*"), which perhaps is a bad interpretation by Hervagius of the written contraction for *Juni* as *Januarii*.
25. In B we read: "... *terminus, vel . v. horae confinium....*" This value gives the best result in the column related to the months of January-December, but I have left the version of A and C because at the moment it appears in the majority.
26. I have translated '*umbilicus*' as 'gnomon' according to the teaching of Pliny the Elder ("*umbilicus, quem gnomonem vocant*"), *Natural history*, VI, 39, 2; this lesson was approved also by Bede ("*Umbilici quem gnomonem appellant*") in *De natura rerum*, *PL*, xc, cap. 48, col. 274A; see also *De temporum ratione*, *PL*, xc, cap. 33, col. 450 A.
27. "first section", that is the order of the hours in the first line. It has to be understood for all the other remaining five 'sections'.
28. Sundials of this type were still in use in the nineteenth century, carved in the hilt of the trip stick of the Tibetan monks. An example can be seen in the Science Museum in London: http://www.sciencemuseum.org.uk/images/I059/10325654.aspx (accessed April 2010).
29. Derek J. de Solla Price, "Review of T. O. Cockayne, ed., *Leechdom, wortcunning and starcraft in early England* [London 1961]", *Journal for the history of medicine*, xvii (1962), 202–3.
30. We are speaking, obviously, of seasonal hours.

31. "*Quando vis scire in dorso astrolapsus hora, in primis scias quantum sol debeat ascendere in ipsa die qua volueris horas probare, et ipsam ascensionem vel altitudinem solis a primo gradu ortus solis usque ad ultimum partire per vi partes ipsasque partes per signa, et, dum sol pervenerit ad ipsa signa in Alhidada, scias sic horas certas usque ad vi; post vi, retorna descendendo usque ad occasum. Sed tu, lector, si diligenter animadvertere quaeris, tu ipse per praedictam walzacoram, id est planant sphaeram diversa poteris fabricare horologia.*" Hermannus Contractus (Gerbertus), *De utilitatibus astrolabii libri duo. Liber primus, caput xxi*: *De inveniendis in dorso astrolabii horis*, *PL*, cxliii, col. 404D.
32. Hermannus Contractus, *De mensura astrolabii liber*, *PL*, cxliii, cols 379–90A.
33. Hermannus Contractus, *De utilitatibus astrolabii libri duo. Liber secundus, caput primum: Demonstratio componendi cum convertibili sciothero horologeci viatorum instrumenti*, *PL*, cxliii, cols 405–8B.
34. He finds the shadow length by a quarter of a circle, divided into 90° on the border. The gnomon (*sciothere*) tip ends in the centre and from there rays reach the outer scale of degrees. At the end of the gnomon a vertical line crosses the degree rays giving the shadow length of that hour.
35. The text is preserved in Ms. Istanbul, Aya Sofya 4830, fol. 129r–v; it is described as a pillar dial construction with a table of vertical shadow lengths. *Cf.* David A. King, *In synchrony with the heavens: Studies in astronomical timekeeping in medieval Islam*, i (Leiden and Boston, 2004), part 4, chap. 7, para. 7.4, pp. 585–6.
36. E. Wiedemann and J. Würschmidt, "Über eine arabische kegelförmige Sonnenuhr", *Archiv für die Geschichte der Naturwissenschaften und der Technik*, vii (1916), 359–76. See also J. Livingston, "The mukhula, an Islamic conical sundial", *Centaurus*, xxvi (1972), 299–313.
37. For a complete description see François Charette, *Mathematical instrumentation in fourteenth-century Egypt and Syria: The illustrated treatise of Najm al-Dīn al-Mīrī* (Leiden, 2003), 145–53.
38. Mario Arnaldi and Karlheinz Schaldach, "A Roman cylinder dial: Witness to a forgotten tradition", *Journal for the history of astronomy*, xxviii (1997), 107–17. Recently a second sundial of this kind has been recognized in France which is dated back to the 3rd–4th century; see Christine Hoët-van Cauwenberghe and Éric Binet, with the participation of Annick Thuet, "Cadran solaire sur os à Amiens (Samarobriva)", *Cahiers du centre Gustave Glotz*, xix (in press) and in *Bulletin des antiquaires de France* (also in press). I thank Dr Christine Hoët-van Cauwenberghe of the University of Lille 3, for telling me of the discovery before their article was published.
39. Gianni Ferrari, "Uno studio sull'orologio romano conosciuto come 'Prosciutto di Portici'", *Gnomonica Italiana*, no. 15 (June 2008), 2–12.
40. The 'cylinder of Este' has two gnomons: one for the summer period (the shorter one, of about 21 mm) and one for the winter period (about 27 mm). My choice of the shorter gnomon is made, of course, because the shadow can easily reach the lower hour point at the summer solstice.
41. The latitude of 45° is that of the 'cylinder of Este'.
42. In the text of the *De utilitatibus astrolabii* this latitude is combined with the seventh climate which is suitable for Britain. "*Climatis septimi latitudo 48 gradus et 32 minuta.... Initium septimi ab Oceano orientali ... partim Franciam majorem, Britanniam, Scotiam, terram Anglicam ...*", Hermannus Contractus, *De utilitatibus astrolabii libri duo*, *PL*, cxlii, chaps. 18 and 19, cols 402D–404B.
43. This is the latitude of Canterbury.
44. The length of the two gnomons is derived from calculation tests because they are now immovable due to oxidization. Schaldach and I have hypothesized lengths of 27mm and 21mm. Maybe the longer one could be between 25 and 27mm.
45. Arnaldi and Schaldach, *op. cit.* (ref. 38), Figs 5 and 6, pp. 113–14.
46. A. L. Binns, on the basis of a passage in the Byrthferth *Manual* and the tables calculated for twelfth-century Iceland by Oddi, prefers to date the object to the first years of the eleventh century; see A. L. Binns Hull, "Sun navigation in the Viking age, and the Canterbury portable sundial", *Acta archaeologica*, xlii (1971), 23–34.
47. The measurements that we use in this section have been made from the photographs of the original

instrument published by Allan Mills, "The Canterbury pendant: A Saxon seasonal-hour altitude dial", *BSS bulletin*, xcv/2 (June 1995), 39–44. D. Jordan furnishes different and more approximate measurements: height 6 cm, width 2.5 cm and the gnomon 2.5 cm (the height could refer to the dial itself without the hanging ring). See D. Jordan and David A. King, *Überlegungen zur Angelsächsichen Sonnenuhr von Canterbury — Reflections on the Canterbury sundial* (Johann Wolgang Goethe-Universität, Institut fur Geschichte der Naturwissenschaften, Preprint series no. 9, 1988), 1–16 and 17–28 respectively.

48. The flat shape of the Canterbury pendant is not to be considered unique; a very similar instrument, a Syrian sundial of the twelfthth century correctly calculated, has been described by Casanova (Paul Casanova, "La montre du sultan Nour ad Din (554 de l'Hégire — 1159–1160)", *Syria*, iv (1923), 282–99). A third testimony is given in the words of de Solla Price, *op. cit.* (ref. 29), where he wrote that he has seen a sample "of less elegance" very similar to the pendant of Canterbury in 1962, at the British Museum, but I admit that I have never seen it.
49. See Derek J. de Solla Price, "Portable sundials in Antiquity...", *Centaurus*, xiv (1969), 242–66; de Solla Price, *op. cit.* (ref. 29); Binns, *op. cit.* (ref. 46); Jordan and King, *op. cit.* (ref. 47); Mills, *op. cit.* (ref. 47); Peter I. Drinkwater, "Comment upon the Canterbury pendant", *BSS bulletin*, xcv/3 (October 1995), 48.
50. On the bottom of the plate of the Canterbury pendant there is a hole that penetrates into the body of the instrument. The gnomon is inserted in this when not in use.
51. Binns, *op. cit.* (ref. 46).
52. Mills, *op. cit.* (ref. 47).
53. This position seems underlined by the head of dragon/snake that in its action of swallowing the sun in its jaws gives us a classical insight into winter iconography.
54. These measures do not seem casual; they are almost identical to those of the gnomons of the cylinder of Este.
55. Charette, *op. cit.* (ref. 37), 145–53.
56. We know almost nothing about this author, only that he came from Sicily (al-Siqillī).
57. 'Stick' is not the correct translation: it would be better as 'tongue'. It is meant to be a movable stick.
58. Wiedemann and Würschmidt, *op. cit.* (ref. 36).
59. This has been demonstrated also by Livingson, *op. cit.* (ref. 36).
60. Schaldach, *op. cit.* (ref. 9) has well underlined the Hellenistic origin of the 'shadow schemes', even if the *terminus post quem* for the sketch to text no. 3 can now be established as the eighth century.
61. See ref. 24.
62. R. Kellog and M. Sullivan, "The Tiberius manuscript horologium", *The compendium*, iv/2 (June 1997), 1–13.
63. See Vitruvius, *De architectura*, I, 6.6; Hygunus Gromaticus, *Constitutio limitum*, chap. 19; Pliny, *Natural history*, II, 76.
64. Although the grave of the Este physician has been dated to the first century, the sundial was probably of the first half of the first century B.C. See Arnaldi and Schaldach, *op. cit.* (ref. 38). The Roman handbooks of gnomonics covered a wide range (unfortunately little has survived) and are attested by Vitruvius, who in chap. 8 of the ninth book of his *De architectura* writes: "*Item ex his generibus viatoria pensilia uti fierent, plures scripta reliquerunt. Ex quorum libris, si qui velit, subiectiones invenire poterit, dummodo sciat analemmatos descriptiones.*"
65. The measures of the shadow lengths of the other hours of the day seem to be found in simple manuals, such as the *Libellus*, with simple folk methods.